

Factorization of Jet Mass Distribution with Small Radius

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We derive a factorization theorem for the jet mass distribution with a given p_T^J for the inclusive process: $h_1 h_2 \rightarrow JX$. When considering the small jet radius approximation, we factorize the scattering cross sections into a partonic cross section, jet fragmentation function and a jet mass distribution function. The decoupled jet mass distributions for a quark and gluon jets are scale invariant, universal and can be extracted from the ratio of two scattering cross sections: $d\sigma/(dp_T^J dM_J)$ and $d\sigma/dp_T^J$. We also obtain resummed results of large logarithms arising from the small jet radius and the jet mass.

Jets created in hadronic colliders contain valuable information to study QCD and beyond. The perturbatively and collimated kinematic features enable us to understand high energy interactions and explore new physics in a robust theoretical way. Towards this end, there have been a lot of studies of jet substructures, which improve our jet identification and characterize boosted heavy particles.

As the energy of collisions increases, it is necessary to employ small radius jets in order to resolve highly energetic particles into multiple jets. This helps us to separate signals we are interested in, such as the production of top quarks or Higgs bosons from the QCD background. When considering small radius jets we can also suppress contaminations arising from the underlying/pile-up events. However, from a theoretical aspect, the small radius limit complicates the perturbative analysis. This limit induces large logarithms in the fixed order calculation in α_s . Hence we have to resum the large logarithms to all orders in order to obtain reliable results.

Jet mass is one of the most important jet substructures. If we can separate the QCD background jets effectively, we can identify a boosted heavy particle from the peak of the jet mass by reconstructing a signal jet. However, a precise description of the jet mass distribution in perturbative QCD is rather subtle. In the small limit of the jet radius R , we can systematically obtain a power counting of the collinear and soft modes and effectively ignore soft gluon emissions with a wide angle from the jet direction. It is also known that the small R approximation practically works well even for the case when R is 0.6 - 1 [1, 2].

In the small R limit a typical scale for the jet mass is comparable to $p_T R$, where p_T is the jet transverse momentum with respect to the beam axis. So if p_T is a few TeV, we can describe the jet mass up to a few hundreds of GeV while employing the small R approximation. However, in the perturbative expansion for QCD jet, the nonzero jet mass distribution is roughly given as α_s/M_J . Thus the region $M_J \ll p_T R$ is dominant and the resummation of the large logarithms such as $\ln(p_T R/M_J)$ is inevitable [2-4].

In this Letter we derive a factorization theorem for the inclusive jet scattering cross section. This allows us to handle large logarithms arising from the small R limit. Then we introduce the normalized jet mass distribution which can be universally applied to any QCD jet with small R . We show that it can be separated from the jet scattering cross section with a given p_T and rapidity. The jet mass distribution turns out to be scale invariant and can describe the peak ($M_J \ll p_T R$) as well as the tail ($M_J \sim p_T R$) regions. However, the peak region needs additional re-factorization to resum the large logarithms properly.

Considering inclusive jet scattering cross section, it can be factorized as

$$\frac{d\sigma}{dy dp_T^J} = \int_{z_J=p_T^J/Q_T}^1 \frac{dz}{z} \frac{d\sigma_i(y, z_J/z)}{dy dp_T^i} D_{J/i}(z). \quad (1)$$

Here σ_i is the cross section with a parton i in the final state and Q_T is the maximal p_T . Since we consider an observed jet in the central rapidity region, the rapidity y scales as $\lesssim \mathcal{O}(1)$. $D_{J/i}$ is the jet fragmentation function (JFF) for a given fragmenting parton i . JFF can be described as the probability that the outgoing jet from the mother parton acquires a momentum fraction z . In order to include the information on the jet mass, we introduce a generic jet function denoted as $\Phi_{J/i}(z, M_J^2)$:

$$D_{J/i}(z) = \int_0^{\Lambda_{\text{alg}}^2} dM_J^2 \Phi_{J/i}(z, M_J^2). \quad (2)$$

Here Λ_{alg}^2 is the maximal jet mass for a given jet algorithm. Thus the differential cross section of the jet mass can be written as

$$\frac{d\sigma}{dy dp_T^J dM_J^2} = \int_{z_J}^1 \frac{dz}{z} \frac{d\sigma_i(y, z_J/z)}{dy dp_T^i} \Phi_{J/i}(z, M_J^2). \quad (3)$$

For a detailed perturbative results of $\Phi_{J/i}$ we consider k_T type algorithm to include recombinational k_T , anti- k_T , and Cambridge/Aachen (C/A), where the jet merging conditions of two particle emissions are given as

$$\theta < R' \begin{cases} R' = R & \text{for } e^+e^- \text{ collider} \\ R' = R/\cosh y & \text{for hadron collider.} \end{cases} \quad (4)$$

Here $\Delta R = \sqrt{\Delta y^2 + \Delta \phi^2}$ for the hadron collider is assumed to be small and can be approximated by $\Delta R \sim \theta \cosh y$. Our result for $\Phi_{J/i}$, using Eq. (4), are applicable to e^+e^- and hadron colliders as well. From our computation, the typical scale for $\Phi_{J/i}$ is given as $p_J^+ \tan(R'/2) \sim E_J R'$. This is expressed as $E_J R$ for e^+e^- annihilation and $p_T^J R$ for hadronic collision.

We will describe $\Phi_{J/i}$ in Eq. (3) in the framework of soft-collinear effective theory (SCET) [5, 6], where collinear and soft modes are separated in a gauge invariant way. Collinear mode in the jet direction for $\Phi_{J/i}$ scales as $p_{n_J} = (p_+, p_\perp, p_-) \sim Q(1, R, R^2)$, where $Q \sim p_T^J$ is a hard scale and R is comparable to the small parameter λ in SCET. In our convention p_\pm are denoted as $p_+ \equiv \bar{n}_J \cdot p = p^0 + \hat{n}_J \cdot \mathbf{p}$ and $p_- \equiv n_J \cdot p = p^0 - \hat{n}_J \cdot \mathbf{p}$, where \hat{n}_J is a unit vector in the jet direction.

Soft modes also contribute to $\Phi_{J/i}$. Therefore, after decoupling soft interactions from the collinear modes of the jet and the beam directions, $\Phi_{J/i}$ can be written in the following convoluted form:

$$\Phi_{J/i}(z, M_J^2; R) = \int dM_c^2 d\ell_- \delta(M_J^2 - M_c^2 - p_J^+ \ell_-) \times J_{J/i}(z, M_c^2; R) \tilde{S}_i(\ell_-; R), \quad (5)$$

where M_c^2 and $p_J^+ \ell_-$ are the collinear and soft contributions to the jet mass (squared) respectively.

Regular soft mode scales as: $p_s \sim Q(\lambda^2, \lambda^2, \lambda^2)$. However the soft contribution to the jet mass is restricted because of phase space constraints in the small R limit. The argument ℓ_- in the soft function is determined from the following factor:

$$\delta(\ell_- - k_-) \Theta(R' - \theta) + \delta(\ell_-) \Theta(\theta - R'), \quad (6)$$

where k is the momentum of a final soft gluon, Θ is a step function, and θ is the relative angle to the collinear parton in the jet. If the soft gluon is included in the jet, the constraint for the phase space is given by

$$R' > \theta \rightarrow \tan^2 \frac{R'}{2} > \frac{k_-}{k_+}. \quad (7)$$

As far as $R \sim \mathcal{O}(\lambda)$, k_- is further suppressed by λ^2 compared to k_+ . So this soft mode, contributing to the jet

mass, should scale as $k \sim Q\lambda^2(1, \lambda, \lambda^2)$.¹ We will call it a “soft-collinear (SC)” mode.

In order to overcome the “scale hierarchy problem” arising due to the small R limit, the separation of SC mode from the full soft mode has been firstly addressed in Refs. [7–9]. We point out that a systematic way to distinguish SC modes is to classify the full soft mode into soft-hard, soft-collinear, and soft-soft modes after decoupling soft interactions from collinear fields. This, in essence, resembles the way of factorizing full QCD degrees of freedom into hard, collinear and soft modes in SCET. Also, and when considering quantum corrections, some overlapped contributions should be eliminated by mechanism similar to “zero-bin subtractions” [10]. More detailed results will be given elsewhere [11].

When applying the constraint, Eq. (6), the contributions from soft-hard and other SC modes with different directions from the one of the jet, vanishes. Because these modes cannot resolve $k_- = n_J \cdot k \sim \mathcal{O}(\lambda^4)$ and regard it as a zero, Eq. (6) for these contributions is just: $\delta(\ell_-)$. As a result, the contributions to the soft function becomes scaleless, i.e., zero. Similar conclusion appears in the consideration of soft dipole interactions in the small R limit [3]. Therefore the soft function in Eq. (5) is described only by n_J -SC mode and it is expressed as

$$\tilde{S}_q(\ell_-) = \frac{1}{N_c} \text{Tr} \langle 0 | Y_{\bar{n}}^\dagger Y_n \delta(\ell_- + \Theta(R' - \theta) \mathcal{P}_s^-) Y_n^\dagger Y_{\bar{n}} | 0 \rangle, \quad (8)$$

where $Y_{n, \bar{n}}$ are SC mode Wilson lines which have the same operator form as the usual soft Wilson lines. We denote $n \equiv n_J$ and \mathcal{P}_s^- is the derivative operator extracting momentum. The soft function initiated by gluons can be also defined similarly in the adjoint representation.

If we consider the jet mass in the full range where $M_J \sim QR \sim \mathcal{O}(\lambda)$, we need only collinear mode scaling as $p_{n_J} \sim Q(1, R, R^2)$. Since SC contribution to the jet mass squared in Eq. (5) scales like $p_J^+ \ell_- \sim \mathcal{O}(\lambda^4)$, then $p_J^+ \ell_-$ can be ignored in the argument of the delta function, and hence the soft function vanishes. Therefore $\Phi_{J/i}$ becomes identical to $J_{J/i}$ in this case.

The collinear jet function, $J_{J/i}$, can be expressed as

$$J_{J_k/q}(z, M^2) = \frac{z^{D-3}}{2N_c} \sum_{X_{\not{q}J}, X_{J-1}} \text{Tr} \langle 0 | \frac{\vec{\eta}}{2} \delta \left(\frac{p_J^+}{z} - \mathcal{P}_+ \right) \delta(M^2 - \Theta(R' - \theta) \mathcal{P}^2) \Psi_n | J(p_J^+, R) X_{\not{q}J} \rangle \langle J(p_J^+, R) X_{\not{q}J} | \bar{\Psi}_n | 0 \rangle, \quad (9)$$

¹ The scaling behavior of SC mode can be expressed in terms of two different small parameters such as $k \sim Q\eta^2(1, \lambda, \lambda^2)$. It might provoke more sub-collinear modes [9]. Setting $\lambda \sim \eta$ in our work, we perform the power counting on M_J and R using one parameter for simplicity.

where k represents a primary parton in the jet, and X_{J-1} denotes final states inside the jet except the primary jet parton. $\Psi_n = W_n^\dagger \xi_n$ is the collinear quark field, where W_n is a familiar collinear Wilson line. This definition, Eq. (9), holds for the jet frame, where $\mathbf{p}_\perp^J = 0$. The

gluon initiated jet function can be similarly defined using $\mathcal{B}_{n\perp}^{a,\mu} = i\bar{n}^\rho g_\perp^{\mu\nu} \mathcal{W}_n^{\dagger,ab} G_{n,\rho\nu}^b$ instead of Ψ_n , where $G_{n,\rho\nu}^b$ is a collinear gluon field strength tensor and \mathcal{W}_n^{ab} is the collinear Wilson line in the adjoint representation.

At tree level the collinear jet function is given as $J_{J_k/i}^{(0)} = \delta(1-z)\delta(M^2)\delta_{ik}$. At next-to-leading order (NLO) in α_s we can divide it into ‘in-jet’ (jet merging) and ‘out-jet’ (jet splitting) contributions, which are proportional to $\delta(1-z)$ and $\delta(M^2)$ respectively. Then we can reorganize the collinear jet function and factorize it into the jet splitting and merging parts in the following manner

$$\begin{aligned} J_{J_k/i} &= J_{J_k/i}^{\text{in}} + J_{J_k/i}^{\text{out}} \\ &= \delta(M^2)\delta(1-z) + \delta(1-z)J_k^{(1)}(M^2) + \delta(M^2)B_{J_k/i}^{(1)}(z) \\ &\sim \left[\delta(1-z) + B_{J_k/i}^{(1)}(z)\right] \left[\delta(M^2) + J_k^{(1)}(M^2)\right] \\ &\equiv B_{J_k/i}(z)J_k(M^2), \end{aligned} \quad (10)$$

where $J_k^{(1)}$ and $B_{J_k/i}^{(1)}$ are in-jet and out-jet contribution at order α_s . It would be interesting to check whether this

factorization still holds to the higher orders in perturbation theory. It is rather unclear because of the presence of multi-gluon interaction Lagrangian and complicated clustering effects.

The jet merging function J_k in Eq. (10) can be identified as a standard jet function when applying the jet algorithm. For example, J_q can be alternatively defined as

$$\begin{aligned} &\sum_{X_n \in J_q} \langle 0 | \Psi_n^\alpha | X_n \rangle \langle X_n | \bar{\Psi}_n^\beta | 0 \rangle \\ &= \int \frac{d^4 p_J}{(2\pi)^3} p_J^+ \frac{\not{n}}{2} J_q(M^2; E_J R') \delta^{\alpha\beta}. \end{aligned} \quad (11)$$

When $M \sim QR$, this jet function’s mass (squared), M^2 , has an upper bound: $\Lambda_{\text{kT}}^2 = p_J^{+2} t^2 / 4$ for kT type algorithm, where $t \equiv \tan(R'/2)$. NLO result for J_q for this case has been computed in Ref. [12]. We also computed the gluon case at NLO. By integrating J_k over M^2 up to Λ_{kT}^2 we also obtain the integrated jet function: $\mathcal{J}_k = \int_0^{\Lambda_{\text{kT}}^2} dM^2 J_k(M^2)$. Then we can interpret $F_k \equiv J_k / \mathcal{J}_k$ as the normalized distributions, and we computed them up to NLO:

$$F_q(M_J^2 \sim Q^2 R^2) = \delta(M_J^2) + \frac{\alpha_s C_F}{2\pi} \left[\frac{1}{M_J^2} \left(-\frac{3}{2}w + 2 \ln \frac{1+w}{1-w} \right) \right]_{\Lambda^2 = \Lambda_{\text{kT}}^2}, \quad (12)$$

$$F_g(M_J^2 \sim Q^2 R^2) = \delta(M_J^2) + \frac{\alpha_s}{2\pi} \left[\frac{N_c}{M_J^2} \left(-\frac{7w}{4} - \frac{w^3}{12} + 2 \ln \frac{1+w}{1-w} \right) + \frac{n_f}{M_J^2} \left(\frac{w}{4} + \frac{w^3}{12} \right) \right]_{\Lambda^2 = \Lambda_{\text{kT}}^2}, \quad (13)$$

where n_f is the number of quark flavors, $w \equiv \sqrt{1 - M_J^2 / \Lambda_{\text{kT}}^2}$, and $[\cdots]_{\Lambda^2}$ is the so-called “ Λ^2 -distribution”, which is defined as

$$\begin{aligned} &\int_0^{\mathcal{M}^2} dM^2 [g(M^2)]_{\Lambda^2} f(M^2) = \\ &\int_0^{\mathcal{M}^2} dM^2 g(M^2) f(M^2) - \int_0^{\Lambda^2} dM^2 g(M^2) f(0). \end{aligned} \quad (14)$$

We also find that the JFF at NLO can be expressed as the multiplication of $B_{J_k/i}$ and \mathcal{J}_k :

$$\begin{aligned} D_{J_k/i}(z) &= \left[\delta(1-z) + B_{J_k/i}^{(1)}(z) \right] \left[1 + \mathcal{J}_k^{(1)} \right] \\ &\sim \delta(1-z) \left[1 + \mathcal{J}_k^{(1)} \right] + B_{J_k/i}^{(1)}(z), \end{aligned} \quad (15)$$

where each function depends on $p_J^+ t$. Details of the perturbative computation of JFF will be shown elsewhere [13].

Next we consider how the generic jet function, $\Phi_{J_k/i}$, can be described in the small jet mass region. In case $M_J \sim Q\lambda^2 \ll QR$, we cannot ignore the soft function

contribution in Eq. (5) since $p_J^+ \ell_- \sim M_J^2$. After redefining the soft function as $S_k(M_s^2) = \tilde{S}_k(\ell_-) / p_J^+$ where $M_s^2 = p_J^+ \ell_-$, the NLO result is given by

$$\begin{aligned} S_k(M^2) &= \delta(M^2) + \frac{\alpha_s C_k}{\pi} \left\{ \delta(M^2) \left(\frac{\pi^2}{24} - \frac{1}{4} \ln^2 \frac{\mu^2 p_J^{+2} t^2}{\Lambda^2} \right) \right. \\ &\quad \left. + \left[\frac{1}{M^2} \ln \frac{\mu^2 p_J^{+2} t^2}{(M^2)^2} \right]_{\Lambda^2} \right\}, \end{aligned} \quad (16)$$

where C_k are C_F for $k = q$ and $C_A (= N_c)$ for $k = g$. The upper limit for M^2 is arbitrarily assigned as Λ^2 which is power counted as $\mathcal{O}(\lambda^4)$. Since $t \equiv \tan(R'/2) \sim \mathcal{O}(\lambda)$, the scale to minimize the large logarithms should be: $\mu \sim \mathcal{O}(Q\lambda^3)$.

Collinear contribution to the jet mass should be also $M_c \sim Q\lambda^2$. So there is a scale hierarchy between in-jet and out-jet contributions for $J_{J_k/i}$ in Eq. (10). While the typical scale for the out-jet contribution, $B_{J_k/i}$, remains as $\mu \sim p_J^+ t \sim \mathcal{O}(Q\lambda)$, the scale for the in-jet contribution should be chosen as $\mu \sim M_c \sim \mathcal{O}(Q\lambda^2)$ for the safe perturbative expansion. In this case the jet merg-

ing function J_k identifies the upper limit of the jet mass, $\Lambda_{\text{KT}}^2 = p_J^+ t^2/4$, as an infinity and ends up as the standard jet function [6] without applying the jet algorithm.

The separation of $J_{J_k/i}$, shown in Eq. (10) can be systematically done in the effective theory approach. After integrating out collinear modes with fluctuations $p^2 \sim Q^2 \lambda^2$ at $\mu \sim Q\lambda$, we obtain the out-jet contributions: $B_{J_k/i}$. Then we compute the in-jet contributions $J_k(M^2 \sim Q^2 \lambda^4)$ after scaling down to $\mu \sim Q\lambda^2$. So in the small jet mass limit the separation of $J_{J_k/i}$ can, in principle, be achieved to any desired order in α_s .

After multiplying/dividing the integrated jet functions J_k we describe $\Phi_{J_k/i}$ in the small jet mass limit as

$$\Phi_{J_k/i}(z, M_J^2 \sim Q^2 \lambda^4) = D_{J_k/i}(z; p_J^+ t) \mathcal{J}_k^{-1}(p_J^+ t) \quad (17)$$

$$\times \int_0^{M_J^2} dM^2 J_k(M^2; \Lambda) S_k(M_J^2 - M^2; \Lambda^2/(p_J^+ t)),$$

where J_k is the standard jet function applying Λ^2 -distribution in Eq. (14) and x in each function with a form $f(y; x)$ represents a scale choice to minimize large logarithms. $\Lambda \sim M_J \sim Q\lambda^2$ is an arbitrary upper limit of the invariant mass for the jet and soft functions, but can be cancelled in the final combined result. Eq. (17) indicates that, especially in case of small R , the problem of resumming nonglobal logarithms such as $\ln M_J/p_T^J R$ can be resolved by an additional factorization, where each factorized function is governed by only one scale.

Finally we have a factorization theorem for the jet mass spectrum given by:

$$\frac{d\sigma}{dy dp_T^J dM_J^2} = \int_{z_J}^1 \frac{dz}{z} \frac{d\sigma_i(y, z_J/z)}{dy dp_T^i} D_{J_k/i}(z) F_k(M_J^2). \quad (18)$$

Here F_k are the jet mass distributions, which are shown in Eqs. (12) and (13) as $M_J \sim p_T^J R$. In the case $M_J \ll p_T^J R$, from Eq. (17), they are expressed as

$$F_k(M_J^2 \sim Q^2 \lambda^4) = H_k(\mu; p_J^+ t) \int_0^{M_J^2} dM^2 J_k(M^2, \mu; \Lambda)$$

$$\times S_k(M_J^2 - M^2, \mu; \Lambda^2/(p_J^+ t)), \quad (19)$$

where $H_k = \mathcal{J}_k^{-1}$. The typical scales for H_k , J_k , and S_k minimizing the large logarithms are given by $\mu \sim Q\lambda$, $Q\lambda^2$, and $Q\lambda^3$ respectively. We call the relevant degrees of freedom of H_k , J_k , and S_k as hard-collinear, collinear, and soft-collinear modes respectively. Eqs. (18) and (19) are our main results.

Note that F_k is a scale invariant quantity because $\sigma_i \otimes D_{J_k/i}$ in Eq. (18) is scale invariant. In the case $M_J \sim p_T^J R$, it is immediate to see that from the perturbative results in Eqs. (12) and (13). We have also confirmed the scale invariance of Eq. (19) by considering the sum of the anomalous dimensions of the factorized functions. The

anomalous dimensions for $k = q$ from NLO results are:

$$\gamma_H^q = -\frac{\alpha_s C_F}{2\pi} \left(2 \ln \frac{\mu^2}{p_J^+ t^2} + 3 \right), \quad (20)$$

$$\gamma_J^q(M^2) = \frac{\alpha_s C_F}{2\pi} \left\{ \delta(M^2) \left(4 \ln \frac{\mu^2}{\Lambda^2} + 3 \right) - \left[\frac{4}{M^2} \right]_{\Lambda^2} \right\},$$

$$\gamma_S^q(M^2) = \frac{\alpha_s C_F}{2\pi} \left\{ -2\delta(M^2) \ln \frac{\mu^2 p_J^+ t^2}{(\Lambda^2)^2} + \left[\frac{4}{M^2} \right]_{\Lambda^2} \right\}.$$

As seen in the factorization theorem in Eq. (18), F_k has been multiplied with $\sigma_i \otimes D_{J_k/i}$, and is not given as a convolution. So by comparing the experimental results of $d\sigma/(dp_T^J dy dM_J^2)$ and $d\sigma/(dp_T^J dy)$, we can extract F_k . This result can be universally applied to any other inclusive jet process. Also, and based on the results of Eqs. (12), (13) and (19), we perturbatively computed F_k with accuracy of next-to-leading logarithms (NLL) plus the fixed NLO in α_s . In order to cover full jet mass range, to the resummed result of Eq. (19), we added the fixed NLO result, $F_k(M_J^2 \sim p_T^{J2} R^2) - F_k(M_J^2 \ll p_T^{J2} R^2)$ at $\mu \sim p_T^J R$. This effectively removes some overlapped contribution between the distributions for the large and small jet mass. For the resummed result of Eq. (19), we solve renormalization group (RG) equations with the anomalous dimensions, $\gamma_{H,J,S}^{q,g}$. And we perform the RG evolutions from the factorization scale of hard-collinear scales for H_k , the collinear for J_k , and the soft-collinear for S_k . After the evolution between the different scales is performed, we checked that the factorization scale dependences cancel.

The default hard-collinear, collinear, and soft-collinear scales have been chosen as $(\mu_{hc}^0, \mu_c^0, \mu_{sc}^0) = (p_T^J R, M_J, M_J^2/(p_T^J R))$, where M_J is a running jet mass variable scaling as $\sim Q\lambda^2$. In order to do safe perturbative expansion and avoid Landau pole as M_J goes to zero, we also introduce a fixed value, M_{cut} . This is similar to the manner which has been introduced in Ref. [14]. Then, in the region $M_J < M_{\text{cut}}$, collinear and soft-collinear scales are fixed as $(\mu_c^0, \mu_{sc}^0) = (M_{\text{cut}}, M_{\text{cut}}^2/(p_T^J R))$. We set M_{cut} as the points where the fixed NLO corrections in the resummed result of Eq. (19) have 10% variation from the local extremum compared to the leading order results. When p_T^J is given as 1.5 TeV and $R = 0.2$, M_{cut} for a quark jet is given by 36.7 GeV and M_{cut} for a gluon jet given by 88.5 GeV. Also we find that the integrated jet mass distributions, $\int dM_J^2 F_k$, start to be stable as ~ 1 from the points we set when M_{cut} varies.

In Fig. 1, with accuracy of NLL+NLO, we show the final results of the jet mass distributions for a quark and gluon jets at $p_T^J = 1.5$ TeV, $R = 0.2$ respectively. These results can be also interpreted as the ratio of two differential cross sections

$$2M_J F_k(M_J^2) = \frac{d\sigma}{dp_T^{Jk} dy dM_J} / \frac{d\sigma}{dp_T^{Jk} dy}, \quad k = q, g. \quad (21)$$

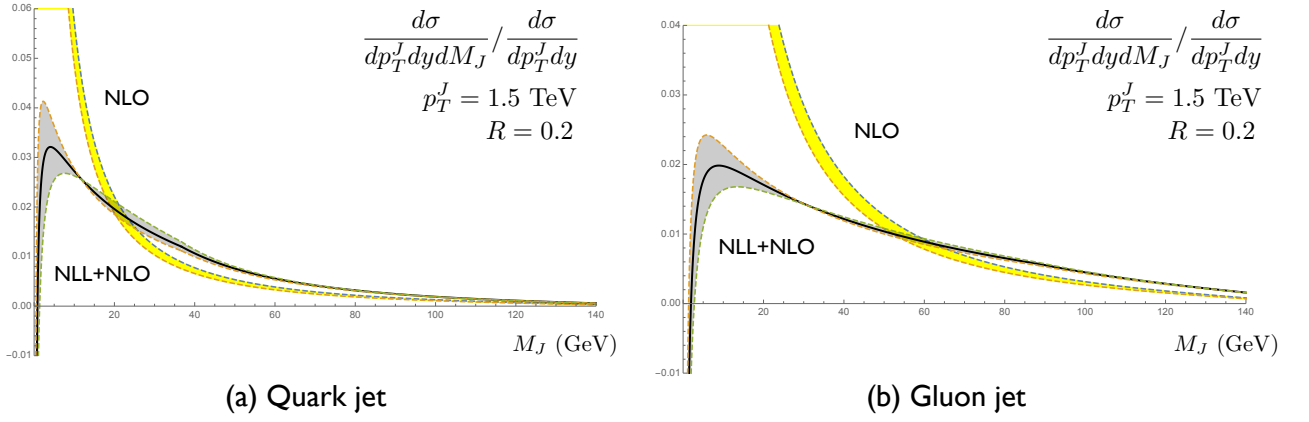


FIG. 1. Jet mass distributions, $2M_J F_k(M_J^2)$, for a quark jet ($k = q$, (a)) and a gluon jet ($k = g$, (b)) with a given $p_T^J = 1.5 \text{ TeV}$, $R = 0.2$. Yellow bands represent fixed NLO result for $2M_J F_k$ for $M_J \sim p_T^J R$.

The black thick lines in Fig. 1 represent our default results with the scale choices of $(\mu_{hc}^0, \mu_c^0, \mu_{sc}^0)$. The bands are the scale variations from $(\mu_{hc}^0, \mu_c^0, \mu_{sc}^0)/2$ to $2(\mu_{hc}^0, \mu_c^0, \mu_{sc}^0)$. The yellow bands denote the large jet mass distributions from Eqs. (12) and (13) with scale variations from $\mu = p_T^J R/2$ to $2p_T^J R$. The jet mass distributions diverge in the small jet mass region, however the NLL resummed distributions show more reliable results in that range according to what was claimed earlier in this Letter. Note that our results in Fig. 1 are obtained purely perturbatively so the results are not reliable in the region $M_J \lesssim 5 \text{ GeV}$, where soft functions are governed by nonperturbative QCD dynamics. In the future, by comparison with experimental data, one can examine how severely nonperturbative interactions affects the jet mass distributions in the small mass region.

To summarize, we present a factorization theorem for the jet mass distribution with a given p_T^J for the inclusive process: $h_1 h_2 \rightarrow JX$. Our results can be also applied to deep inelastic scattering, e^+e^- annihilation and the single jet production: $h_1 h_2 \rightarrow J + L$, where L is a color singlet particle.² The decoupled jet mass distributions, $F_{q,g}$ are universal and promises a consistent treatment for examining QCD jet mass distributions in various processes. More precise estimations of $F_{q,g}$, while including the higher order perturbative results and nonperturbative effects, are needed to clarify QCD jet substructures.

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² While preparing this paper, there appeared Ref. [15], where the jet mass distributions for $h_1 h_2 \rightarrow J + L$ with a jet veto has been considered in detail. In case of small R similar idea of factorization has been applied unless we consider a jet veto. The main

difference of our work from Ref. [15] is that while considering an observed jet transverse momentum, we systematically introduced JFF for explaining the jet splitting. More studies are needed for a detailed comparison.